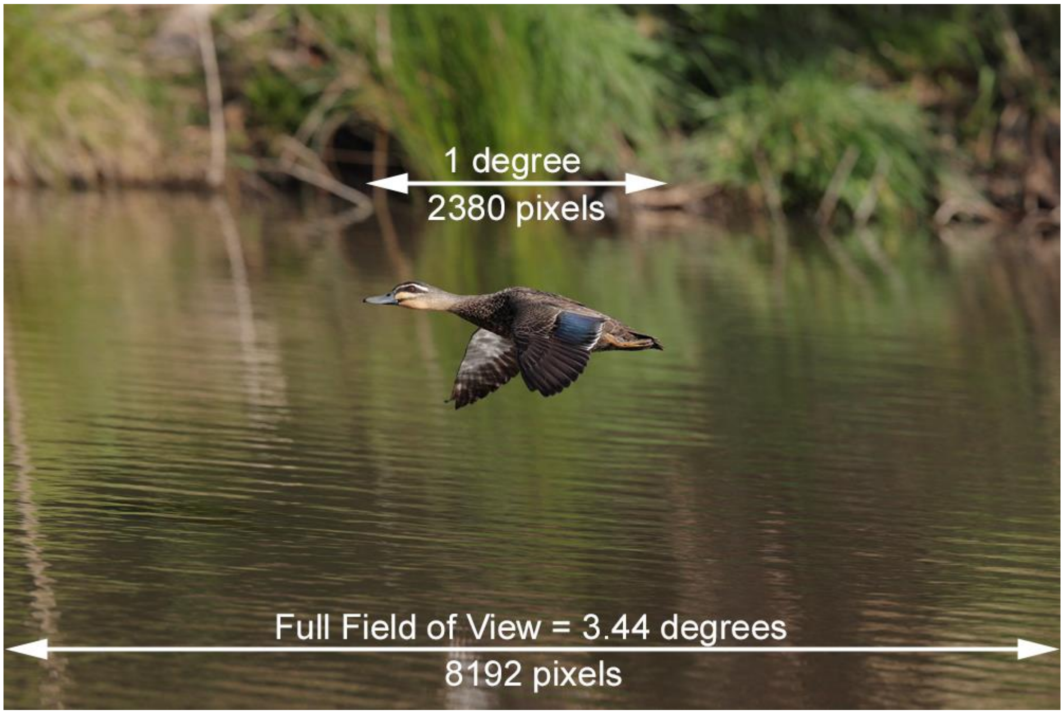
Reach and Cropped Sensor Myths – Ian Wilson PhD(optics)

The purpose of this note is to try to explain the meaning of the photography term 'reach' and to debunk some myths regarding cropped sensor cameras and their full-frame equivalent. The idea of full-frame equivalence is widely misunderstood and can lead to poor gear choices by novices and the unwary.

A common myth is that a cropped sensor camera has more reach than a full-frame camera. To test this myth, we need to clarify what photographers mean by reach. It is an ill-defined term related to how close the subject captured in the digital image appears to be; increasing reach will make the subject appear bigger and closer to the viewer. This effect can only be produced by increasing the focal length of the lens and/or by reducing the size of the sensor pixels. In either case the digital image will have more pixels on the subject. In our branch of wildlife photography, the subject will be a bird which leads to the useful idea of 'pixels on the bird' (POB). The greater the reach the more POB and the closer the bird will appear to be when the digital image is viewed. POB can be measured and can therefore be used as a convenient way to quantify reach. Armed with this concept we can now confidently assert that if two different camera systems, for example a cropped sensor and a full-frame system have the same number of POB, then they must have the same reach. An example of this is the cropped sensor Canon 7DII + 600 mm f/4 and full-frame Canon 5Ds + 600 mm f/4; both cameras have sensors with 4.1 µm (0.0041 mm) pixels and both systems have the same lens focal length (600 mm). The size of the bird in the optical image projected on the sensor by the lens will be the same in each case and as the sensor pixel size is also the same, it follows that the number of POB will be the same. What is different is the full field of view with the full-frame image having more space around the bird but when viewed 1:1, that is 100%, the bird will appear to be the same size in each case and hence the reach is the same for both systems.

A small part of the field of view is recorded by each sensor pixel and if we add together all the small angular parts, we get a value for the full field of view. A basic principle of geometric optics tells us that the small angular field of view imaged by a single sensor pixel is $\delta\theta$ = pixel size ÷ focal length. $\delta\theta$ is a symbol commonly used in science to indicate a small number in angle measure (radian). In the present case it is a measure of the angular resolution of the system and in our example will be $\delta\theta = 0.0041 \div 600 = 0.00000683$ radian. The Canon 7DII has 5472 pixels across the width of the sensor so the width of the full field of view with a 600 mm lens will be $5472 \times 0.0000068 = 0.0374$ radian which in degrees is an angle of 2.142°. The conversion from radian measure to degrees uses the fact that π radian = 180°. The Canon 5Ds has 8688 pixels across its sensor width so the full field of view is $8688 \times 0.00000683 = 0.0593$ radian or an angle of 3.400°. With this insight we now have a quantitative way of measuring reach which will be proportional to the inverse of the angular resolution and conveniently measured in units of pixels per degree. This is the number of pixels corresponding to one degree of the field of view. For the Canon 7DII example the reach will be $5472 \div 2.142 = 2555$ pixels per degree. Similarly, for the Canon 5Ds example, the reach will be $8688 \div 3.40 = 2555$ pixels per degree. In both examples, if the bird covered 1° of the field of view the number of POB would be 2555 pixels. A further example is illustrated in the image below captured with the Canon R5 + 600 mm f/4 III lens. The full field of view is covered by 8192 sensor pixels and spans 3.44°. The duck in the image covers about 1° of the field of view which for this set-up is 2380 sensor pixels. The reach is therefore 2380 pixels /degree.



To complete this discussion, we return to the beginning where we wrote that reach can only be increased by increasing the focal length and/or by reducing the size of the sensor pixels. Reach has nothing to do with the crop factor – myth busted. To further illustrate this point, we have drawn up the table below which shows the reach in pixels/degree for current Canon camera models mounted with lenses ranging from 400 mm to 840 mm focal length.

Canon Camera	Pixel Size (µm)	Lens 400 mm	Lens 500 mm	Lens 560 mm	Lens 600 mm	Lens 700 mm	Lens 800 mm	Lens 840 mm
90D	3.2	2182	2727	3054	3272	3818	4363	4581
7DII	4.1	1702	2128	2384	2554	2980	3405	3576
5Ds	4.1	1702	2128	2384	2554	2980	3405	3576
R5	4.4	1587	1983	2221	2380	2777	3173	3332
5DIV	5.4	1293	1616	1810	1939	2262	2586	2715
R6	6.6	1058	1322	1481	1587	1851	2116	2221
1DxIII	6.6	1058	1322	1481	1587	1851	2116	2221

Notice that the reach of the Canon R5 + 400 mm lens is the same as the flagship Canon 1DxIII + 600 mm lens and the reach of the Canon 90D + 400 mm lens is almost as great as the Canon 1DxIII + 840 mm lens. It means that the Canon 90D + 400 mm lens will have almost as many POB as the Canon 1DxIII + 840 mm lens. This does not mean that the digital image quality of these two systems will be comparable; final image quality depends on more than the simple geometric optics used to quantify the reach. To a first approximation the optical image quality will be similar for top of the line Canon lenses but there will be a noticeable difference in the digital image quality due to the different digital noise, colour bit depth and dynamic range of the two sensors.

This is a good point in the discussion to consider crop factors, in particular, the often cited full-frame equivalency of cropped sensor cameras. First, we need to explain the meaning of the term crop factor. Crop factor is the scaling multiplier required to scale up the size of a cropped sensor system to that of a full-frame system. It is easy to work out the crop factor if the cropped sensor size is known. For example, the Canon 7DII has a sensor that is 22.4 mm wide by 15 mm high; a full-frame sensor is 36 mm by 24 mm, so the crop factor required to scale up the Canon 7DII sensor to full frame size based on the sensor width is 36 ÷ 22.4 = 1.61 and 24 ÷ 15 = 1.6 based on the sensor height. This small difference in crop factor is neither here nor there and the accepted value of the crop factor for this camera is 1.6.

Scaling is a fundamental principle in optical systems design governed by the following rule. Design parameters with dimensions measured in units of length, such as focal length, overall length of the lens and the lens diameter, all scale in proportion. When a sensor is included in the system its dimensions also scale in the same proportions as the lens. Notably, the sensor width and height (mm) and sensor pixel size (µm), both with dimensions measured in units of length, scale in the same proportions as the lens design. Other design parameters that are dimensionless, such as ratios and angular measures, do not change with scaling and are said to be scaling invariant. Parameters that are scaling invariant include refractive index, f/No, field of view, distortion, and relative illumination. The principle of scaling makes it possible to calculate a new optical design based on an existing design. This principle is often used by lens designers when they want a starting design for a new lens. They select a tried and tested design and rescale it to the new focal length that they require. We use crop factors in the same way; the crop factor is the scaling multiplier required to get to the full-frame equivalent camera system.

Here is an example of how a cropped sensor camera is scaled to the full-frame equivalent camera system. Let us start with the Canon 7DII cropped sensor mounted with a 400 mm f/4 lens. As shown earlier, the crop factor is 1.6. Here is what the full-frame equivalent looks like with a 1.6× scaling factor:

	Cropped Sensor System	Full-frame Equivalent System
Effective Focal Length (mm)	400	400 × 1.6 = 640
Sensor Size (mm)	22.4 × 15	36 × 24
Sensor Pixel Size (µm)	4.1	4.1 × 1.6 = 6.56
Number of Sensor Pixels (Mpx)	20.2	20.2
F/No	f/4	f/4
Full Field of View (degrees)	3.21	3.21
Reach (pixels/degree)	1702	1702

Note that the number of megapixels, f/No, field of view and reach remain the same, they are scaling invariant. The closest full-frame equivalent camera system in the Canon line-up is the Canon R6 + 600 mm f/4 or Canon 1DxIII + 600 mm f/4. The optical image quality of the cropped and full-frame equivalent systems will be comparable, the reach and resolution will be similar, and in good light with a low ISO the digital image quality viewed 1:1 will also be comparable. However, in average and poor light, the signal to noise ratio, colour bit depth and dynamic range of the full-frame equivalent systems will be better and as a result the digital image quality will also be better.

Micro-4/3 cameras are a more extreme example with a crop factor of 2. Some BLP members use these camera systems because they are small, light-weight and cost effective. A popular combination is the Olympus OM-D E-M1 series with the M.Zuiko 300 mm f/4 lens. The full-frame equivalent system looks like the following:

	Cropped Sensor System	Full-frame Equivalent System
Effective Focal Length (mm)	300	300 × 2 = 600
Sensor Size (mm)	17.4 × 13	34.8 × 26
Sensor Pixel Size (μm)	3.35	3.35 × 2 = 6.7
Number of Sensor Pixels (Mpx)	20.4	20.4
F/No	f/4	f/4
Full Field of View (degrees)	3.32	3.32
Reach (pixels/degree)	1563	1563

As in the previous example, the optical image quality of the micro-4/3 system and the full-frame equivalent will be comparable with similar reach and resolution. However, there will be a noticeable difference in the digital image quality when viewed 1:1, most notably in the digital noise and consequent loss of contrast in fine detail. The main source of digital noise in modern camera sensors is photon shot noise due to the random arrival of photons at the sensor surface even when uniformly illuminated with light. This leads to the appearance of digital noise in the image which is easy to see in brighter smoothly varying background areas. Photon shot noise is not caused by the sensor design or associated electronics but is a fundamental property of light. The amount of photon shot noise in an image is determined by the area of the sensor pixel; larger pixels have less noise than smaller pixels. The amount of shot noise scales as the square-root of the pixel area. In the micro-4/3 example the area of the pixel is 3.35 × 3.35 = 11.2 square μm. The full-frame equivalent system has pixels with area 6.7 × 6.7 = 45 square μm or four times the area of the cropped sensor pixels. The amount of photon shot noise in the full-frame equivalent system will therefore be less by a factor of square-root 4, that is half the amount of noise in the micro-4/3 image. This is a significant difference which will be noticeable when viewing images at 1:1 scale. Photon shot noise appears in the image as graininess and has an effect like dithering which blurs edges of small tonal difference and reduces the contrast of fine detail. It means that micro-4/3 images usually need more noise reduction and sharpening to recover the low contrast fine details in the image.

The main factor determining how much recoverable detail is recorded in the dark parts of the image is the dynamic range of the sensor. Unlike photon shot noise, dynamic range depends on the design of the sensor. Sensor testing websites like DxOMark and Photons to Photos provide measurements of the dynamic range as a function of ISO with convenient comparison tools. The tools can be used to compare the dynamic range of micro-4/3 sensors with full-frame equivalent sensors. For the latest Olympus OM-D E-M1 MkIII and the flagship OM-D E-M1X, the dynamic range is up to 1 EV less than full-frame equivalent cameras of the same generation; this is a significant difference. Dynamic range is also an important metric used to quantify digital image quality. For acceptable image quality the dynamic range must be above a certain threshold value. The ISO at this threshold value is the maximum ISO that will provide acceptable image quality without the need for special noise reduction and sharpening. The Photons to Photos websites shows that the high ISO limit for the latest Olympus OM-D E-M1 sensors is about ISO 2400 which is less than half the high ISO limit of equivalent full-frame sensors.

In summary, we have shown that rescaling a cropped sensor camera system to an equivalent full-frame system has questionable validity. The two systems are only equivalent in terms of the basic optical image properties. When we compare a digital image at 1:1 scale produced by a cropped sensor system and its full-frame equivalent there is a significant difference in image quality for large crop factors, and it is misleading to suggest there is an equivalence in quality. That is not to say micro-4/3 systems do not have a place in bird photography, it is more of a warning to unsuspecting photographers who 'buy' the marketing hype that these systems are giant killers. They have a place because they are small, light-weight and cost effective but to achieve these desirable features the digital image quality has been compromised. For photographers wanting the best image quality under a wide range of conditions, full-frame systems always have been and always will be the best choice.